

Solutions to Mathematics Workbook-1 | Quadratic Equations & Inequalities [2022]

Level - 2	Daily Tutorial Sheet 9
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106.(B) $\alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$

$\gamma + \delta = \frac{-q}{p}, \gamma\delta = \frac{r}{p}$

It is given that $\alpha, \beta, \gamma, \delta$ are in AP. $\Rightarrow \beta - \alpha = \delta - \gamma$

$\Rightarrow (\beta - \alpha)^2 = (\delta - \gamma)^2 \Rightarrow (\beta + \alpha)^2 - 4\alpha\beta = (\delta + \gamma)^2 - 4\delta\gamma$

$\Rightarrow \frac{b^2}{a^2} - \frac{4c}{a} = \frac{q^2}{p^2} - \frac{4r}{p} \Rightarrow \frac{b^2 - 4ac}{a^2} = \frac{q^2 - 4rp}{p^2} \Rightarrow \frac{D_1}{D_2} = \frac{a^2}{p^2} \quad \left\{ \begin{array}{l} \text{Using } D_1 = b^2 - 4ac \\ \text{and } D_2 = q^2 - 4rp \end{array} \right\}$

107.(A) Let D_1 and D_2 be the discriminants of $x^2 + b_1x + c_1 = 0$ and $x^2 + b_2x + c_2 = 0$ respectively.

This implies $D_1 = b_1^2 - 4c_1$ and $D_2 = b_2^2 - 4c_2$

Now check $D_1 + D_2 = b_1^2 + b_2^2 - 4(c_1 + c_2)$

$= b_1^2 + b_2^2 - 2(b_1b_2) \quad \{\text{using } b_1b_2 = 2(c_1 + c_2)\}$

$= (b_1 - b_2)^2 \geq 0$

$\Rightarrow$ At least one of D_1 and D_2 is positive as $D_1 + D_2 \geq 0$

$\Rightarrow$ Either $x^2 + b_1x + c_1 = 0$ or $x^2 + b_2x + c_2 = 0$ or both has real roots $\Rightarrow$ at least one equation has real Roots

108.(C) α, β are roots of $ax^2 + bx + c = 0$, $\alpha + \beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$

The given equation is $x^2(a - b + c) + x(b - 2c) + c = 0$

Sum of roots $= \frac{-(b - 2c)}{a - b + c} = \frac{-b + 2c}{a - b + c} = \frac{\frac{-b}{a} + \frac{2c}{a}}{1 + \frac{b}{a} + \frac{c}{a}} = \frac{\alpha + \beta + 2\alpha\beta}{1 + (\alpha + \beta) + \alpha\beta} = \frac{\alpha(\beta + 1) + \beta(\alpha + 1)}{(1 + \alpha)(1 + \beta)} = \frac{\alpha}{\alpha + 1} + \frac{\beta}{\beta + 1} \dots (i)$

Product of roots is $\frac{c}{a - b + c} = \frac{c/a}{1 - \frac{b}{a} + \frac{c}{a}} = \frac{\alpha\beta}{1 + \alpha + \beta + \alpha\beta} = \frac{\alpha}{\alpha + 1} \cdot \frac{\beta}{\beta + 1} \dots (ii)$

Combining (i) and (ii), roots of the given equation are $\frac{\alpha}{\alpha + 1}$ and $\frac{\beta}{\beta + 1}$

109.(C) Let $f(x) = (1 - a)x^2 + 3ax - 1$

We will use following result in this question

[if a, b are roots of $px^2 + qx + r = 0$. then both a and b positive if :

$pf(0) > 0$, $x_v = x$ -coordinate of vertex > 0 and $D \geq 0$]

$pf(0) = (1 - a)f(0) = (1 - a)(-1) = a - 1 > 0 \quad [\text{it is given that } a > 1]$

$\Rightarrow pf(0) > 0 \quad \dots (i)$

$$x_v = \frac{-3a}{2(1-a)} = \frac{3a}{2(a-1)} \Rightarrow x_v > 0 \forall a > 1 \dots (ii)$$

Let us check discriminant now $D = 9a^2 + 4(1-a) = 9a^2 - 4a + 4$

D is always positive as coefficient of $a^2 > 0$ and discriminant of the quadratic in $a = -128 < 0$

$$\Rightarrow D > 0 \forall a \in R \dots (iii)$$

Combining (i), (ii) and (iii) both roots of the given quadratic equation are positive $\forall a > 1$

110.(B) Let given polynomial be $h(x) = (ax^2 + bx + c)(ax^2 - dx - c)$

Further Let $f(x) = ax^2 + bx + c$ and $g(x) = ax^2 - dx - c$

$$\text{Discriminant of } f(x) = D_1 = b^2 - 4ac \dots (i)$$

$$\text{Discriminant of } g(x) = D_2 = d^2 + 4ac \dots (ii)$$

$$ac \neq 0 \Rightarrow ac > 0 \Rightarrow D_2 > 0 \text{ Or } ac < 0 \Rightarrow D_1 > 0$$

we can say atleast one of D_1 and D_2 would be positive.

$$\Rightarrow \text{at least one of } f(x) = 0 \text{ and } g(x) = 0 \text{ would have real roots. } \Rightarrow h(x) = 0 \text{ has at least 2 real roots.}$$

111.(A) Since the quadratic equation has non-real roots, its graph always remain either above or below X-axis depending on $a > 0$ or $a < 0$ respectively.

$$4(a+b) - 3c > 0 \quad (\text{Given})$$

$$\Rightarrow a(2^2) + 2b(2) - 3c > 0$$

$$\Rightarrow \text{The given quadratic function } > 0 \forall x \in R$$

At $x = 0$, also given quadratic function > 0

$$\Rightarrow -c > 0 \Rightarrow c < 0$$

112.(A) Given $a(p+q)^2 + 2bpq + c = 0$ and $a(p+r)^2 + 2bpr + c = 0$

Above two equations are symmetrical in q and r. $\Rightarrow q$ and r satisfy the equation $a(p+x)^2 + 2bpx + c = 0$

$$\Rightarrow qr = \frac{ap^2 + c}{a} = p^2 + \frac{c}{a}$$

113.(B) Let $f(x) = x^2 + bx + c$

$$f(0) = c$$

As coefficient of x^2 is positive and $f(0) = c < 0$, we have

$$af(0) < 0 \quad [\text{where } a \text{ is coefficient of } x^2]$$

$$\Rightarrow \text{Roots of } f(x) = 0 \text{ are of opposite sign } \Rightarrow [\because \alpha < 0 < \beta] \dots (i)$$

$$\text{Sum of roots} = \alpha + \beta = -b < 0 \quad [\because b > 0]$$

$$\Rightarrow \alpha + \beta < 0 \Rightarrow \beta < -\alpha$$

$$\Rightarrow \beta < |\alpha| \quad [\because -\alpha \text{ is positive}] \dots (ii)$$

Combining (i) and (ii), to get $\alpha < 0 < \beta < |\alpha|$

114.(A) Given equation is $\left(\frac{9}{10}\right)^x = -3 + x - x^2$

Let us consider right hand side of the equation as $\text{RHS} = -3 + x - x^2$

RHS is a quadratic expression with negative coefficient of x^2

It means it will possess maximum value $= \frac{-D}{4a}$ i.e. (y coordinate of vertex)

$$\text{Max \{RHS\}} = \frac{-D}{4a} = \frac{-(b^2 - 4ac)}{4a} = \frac{-(1 - 12)}{4(-1)} = \frac{-11}{4}$$

As maximum value of RHS is negative, RHS will be negative for all $x \in R$

$$\Rightarrow \text{RHS} < 0 \forall x \in R \quad \dots (i) \quad \text{LHS} = \left(\frac{9}{10}\right)^x$$

From graph of a^x ($0 < a < 1$), it is obvious that $a^x > 0 \forall x \in R$

$$\Rightarrow \text{LHS} > 0 \forall x \in R \quad \dots (ii)$$

Combining (i) and (ii), we can say LHS is always positive and RHS is always negative.

Hence there exists no value of x that will satisfy the given equation.

115.(B) α_1 and β_1 are roots of $a_1x^2 + b_1x + c_1 = 0$... (i)

Now we find an equation whose roots are reciprocal of the roots of the above quadratic equation.

i.e. find an equation whose roots are $\frac{1}{\alpha_1}$ and $\frac{1}{\beta_1}$

To find an equation with roots $\frac{1}{\alpha_1}$ and $\frac{1}{\beta_1}$, replace $x = \frac{1}{y}$ in (i) to get,

$$\frac{a_1}{y^2} + \frac{b_1}{y} + c_1 = 0 \quad \{\because \text{roots are reciprocal}\} \Rightarrow c_1y^2 + b_1y + a_1 = 0$$

This equation has both roots common to $a_2x^2 + b_2x + c_2 = 0$

Hence (B) is correct answer.